Research Progress

Modified Bayesian Method for Estimating Directional Wave Spectra from HF Radar Backscatter

九州大学

by

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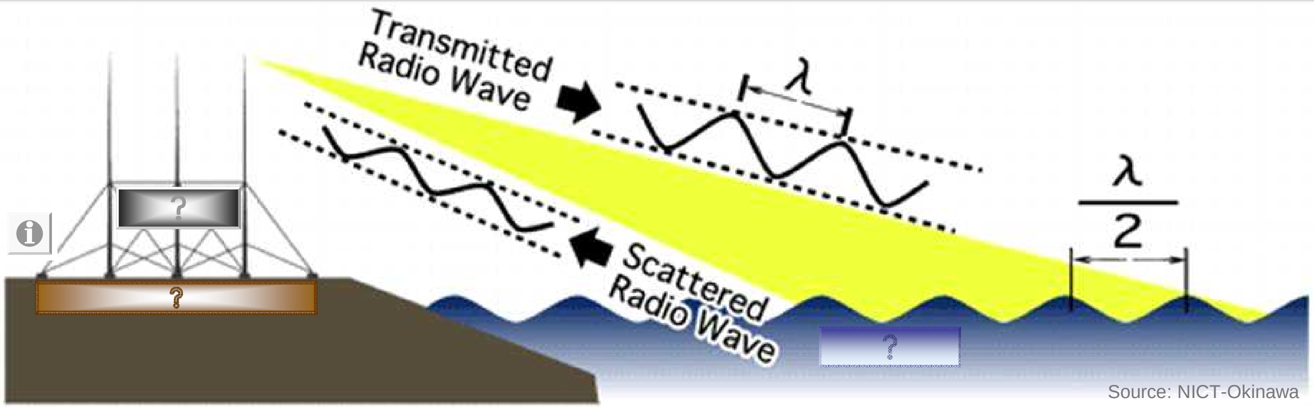
Introduction (1)

The waves in ocean are multidirectional, and information about the directional distribution of wave energy is often required for the design of coastal and offshore structure

Sea surface current, ocean wave spectra and surface wind directions can be measured through frequency analysis of the radio wave backscattered from the ocean surface waves

Schematic diagram of observation using HF ocean surface radar



The shore based radar transmits HF radio waves toward the sea surface and the radio waves are strongly backscattered by ocean surfaces waves with half the radio wave length (*Bragg resonant scattering*)

Introduction (2)

Theoretically, the directional wave spectra can be estimated from 1st and 2nd order Doppler spectra of HF radio wave with respect to the wave spectrum

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A method for estimating directional wave spectra from the data obtained by HF radar has not been established yet

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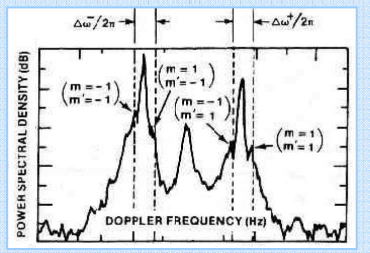
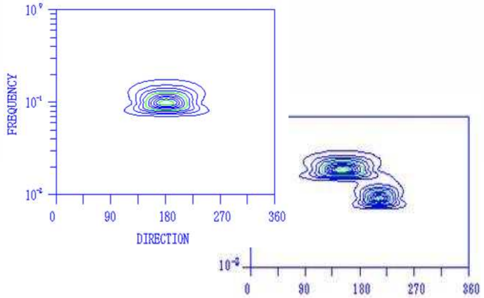
to develop the practical method for estimating directional wave spectra from HF backscattered information

$\hat{S}(f,\theta)$ → Barrick`s equation → $\sigma^{(2)}(\omega)$



Measurement

Actual world



An Identical Twin Experiment

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Bayesian Method ⟷ ?

Fundamental Equations (1)

Doppler Spectrum obtained from HF radar (Barrick, 1972):

$$\text{HF} \quad \sigma(\omega) \approx \sigma^{(1)}(\omega) + \sigma^{(2)}(\omega) \qquad (1)$$

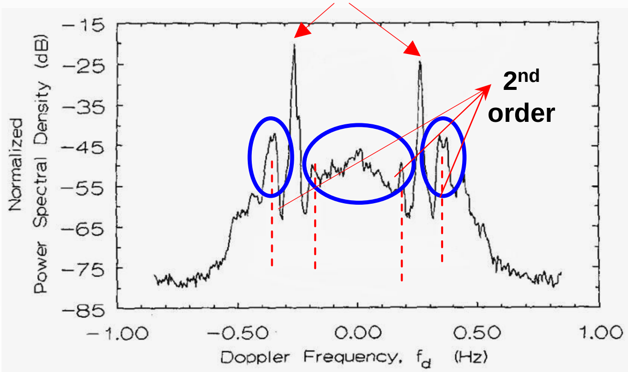
Crombie (1955) discovered that some HF signals recorded near the ocean had a Doppler shift due to *"Bragg scattering"*

First Order Component:

$$\sigma^{(1)}(\omega) = 2^6 \pi k_0^4 \sum_{m=\pm 1} S(-2m\mathbf{k}_0, 0)\, \delta(\omega_{HF} - m\omega_B) \qquad (2)$$

- angular frequency (ω, HF)
- S: wave spectrum
- $\mathbf{k}_0$: absolute value of wave number vector k_0
- ω_B: Bragg angular frequency, $\omega_B = \sqrt{2gk_0}$

1st order



wave number vector k_1 & k_2

$$\mathbf{k}_1 = (p - k_0, q)$$
$$\mathbf{k}_2 = (-p - k_0, -q)$$

$$\mathbf{k}_1 + \mathbf{k}_2 = -2\mathbf{k}_0$$

Bragg's resonance condition

$$\sigma^{(2)}(\omega) = 2^6 \pi k_0^4 \sum_{m_1 m_2 = \pm 1} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\Gamma|^2 S(m_1\mathbf{k}_1) S(m_2\mathbf{k}_2) \times \delta\left(\omega - m_1\sqrt{gk_1} - m_2\sqrt{gk_2}\right) dp\,dq \qquad (3)$$

- dp: parallel to axis radar beam
- dq: parallel to orthogonal radar beam

Second Order Component:

Fundamental Equations (2)

scattering effect

The coupling coefficient is given:

$$\Gamma = \left|\Gamma_{\mathrm{H}} + \Gamma_{\mathrm{E}}\right| \quad (4)$$

coupling Hydrodynamic

$$\Gamma_{\mathrm{H}} = \frac{-i}{2}\left[k_1 + k_2 - \frac{\left(k_1 k_2 - \mathbf{k}_1 \cdot \mathbf{k}_2\right)\left(\omega^2 + \omega_B^2\right)}{m_1 m_2 \sqrt{k_1 k_2}\left(\omega^2 - \omega_B^2\right)}\right] \quad (5)$$

where:

$\omega_B =$ Bragg angular frequency

$\mathbf{k}_1 \& \mathbf{k}_2 =$ wave number vector

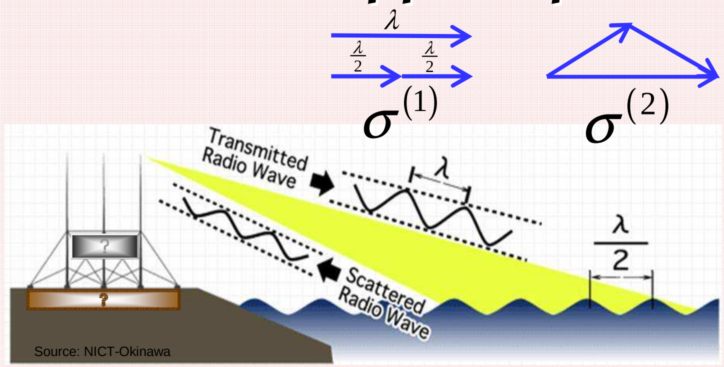
coupling Electro Magnetic

$$\Gamma_{\mathrm{EM}} = \frac{1}{2}\left[\frac{\left(\mathbf{k}_1 \cdot \mathbf{k}_0\right)\left(\mathbf{k}_2 \cdot \mathbf{k}_0\right)/k_0^{\,2} - 2\mathbf{k}_1 \cdot \mathbf{k}_2}{\sqrt{\mathbf{k}_1 \cdot \mathbf{k}_2} - k_0 \Delta}\right] \quad (6)$$

Here Δ refers to the normalized surface impedance derived by Barrick (1972). The absolute value of which is small enough → negligible

Fundamental Equations (3)

Normalized Doppler Spectrum



PROBLEMS ?

The integral is restricted to being solved only on specific lines defined by δ

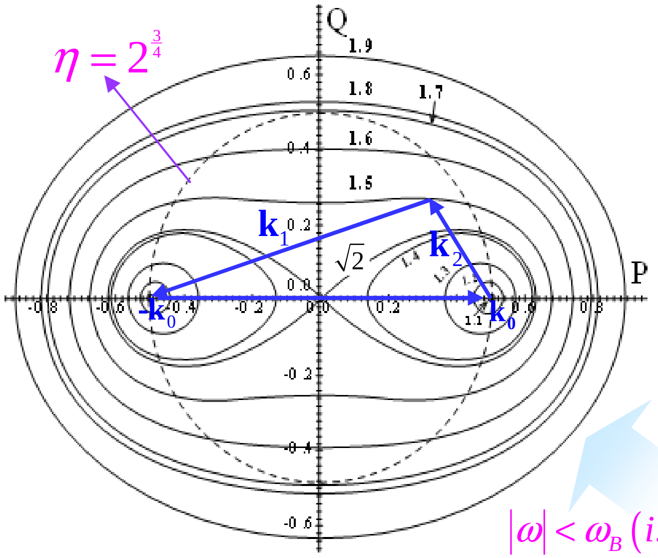
Thus it must satisfy:

$$\omega - m_1\sqrt{gk_1} - m_2\sqrt{gk_2} = 0$$

The digitization of this equation is complicated (incomplete inverse problem) →
number of unknown parameters >> that of equations

$(\eta > 1.0)(\omega > \omega_B)$

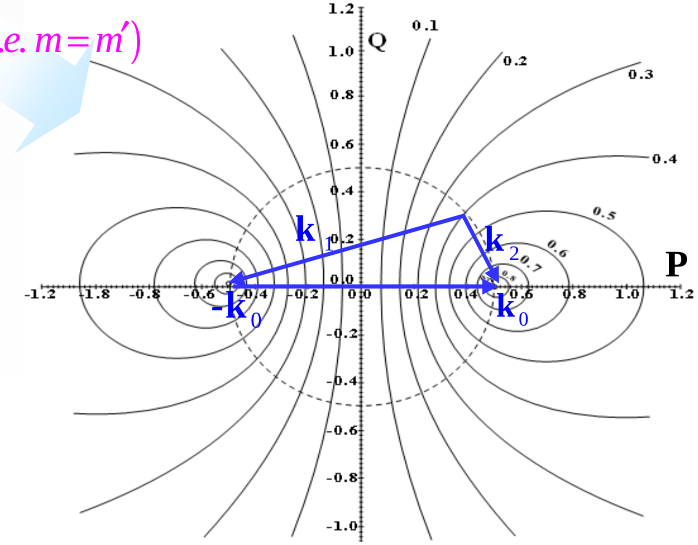
$\eta = 2^{\frac{3}{4}}$



$|\omega| < \omega_B \, (i.e. \, m = -m')$

$(\eta < 1.0)(\omega < \omega_B)$

$|\omega| > \omega_B \, (i.e. \, m = m')$



Fundamental Equations (4)

$$\sigma^{(2)}(\omega) = 2^6 \pi K_0^4 \sum_{m_1 m_2 = \pm 1} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\Gamma|^2 S(m_1\mathbf{k}_1) S(m_2\mathbf{k}_2) \times \delta\left(\omega - m_1\sqrt{gk_1} - m_2\sqrt{gk_2}\right) dp\,dq \quad (3)$$

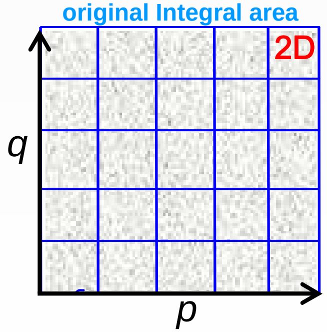
HF ($\sigma^{(2)}(\omega)$, ω); wave ($m_1\sqrt{gk_1}$, $m_2\sqrt{gk_2}$); wave number vector ($\mathbf{k}_1$, $\mathbf{k}_2$); delta function (δ); **2D**

For convenience, the parameters are nondimensionalized by ω_B and $2k_0$ as follows: (7)

$$\tilde{\omega} = \omega / \omega_B \qquad \tilde{\mathbf{k}} = \mathbf{k} / (2k_0) \qquad \tilde{\Gamma} = \Gamma / (2k_0) \qquad \tilde{S}(\tilde{\mathbf{k}}) = (2k_0)^4 S(\mathbf{k})$$

The integration 2nd order with respect to the p & q can be transformed into **a single variable** since the integrand include δ. (**Lippa & Barrick, 82**) ⇒ if the wave propagation direction θ_1 of the wave number vector $\mathbf{k}_1$ is adopted as a single independent variable for the integration

original Integral area

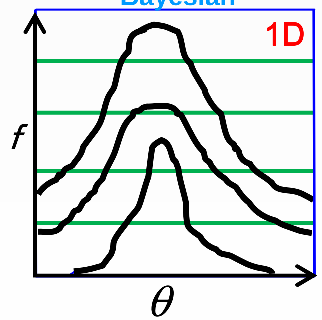


Theoretically

$$\tilde{\sigma}^{(2)}(\tilde{\omega}) = \int_0^{\theta_L} G(\theta_1, \tilde{\omega})\, d\theta_1 \quad (8)$$

1D

Bayesian



where

$$G(\theta, \tilde{\omega}) = 16\pi \left[|\tilde{\Gamma}|^2 \left\{ \tilde{S}(m_1\tilde{\mathbf{k}}_1)\tilde{S}(m_2\tilde{\mathbf{k}}_2) + \tilde{S}(m_1\tilde{\mathbf{k}}_1 *)\tilde{S}(m_2\tilde{\mathbf{k}}_2 *) \right\} y^3 \left| dy/dh \right| \right]_{y=\hat{y}} \quad (9)$$

$$S(\mathbf{k}) = \frac{g^2}{2^5 \pi^4 f^3} S(f, \theta) \quad (12)$$

wave number spectrum ($S(\mathbf{k})$); directional wave spectrum ($S(f,\theta)$)

Recent Researches

Efforts to extract HF radar for directional wave spectra

No	References	Results
1	Wyatt (1990)	Relaxation method → assumed wave spectral form at HF & the nonlinear integral equation is modified into a linear integral equation which can be solved by this meyhod
2	Hisaki (1996)	Development of linearization method → to solve the nonlinear integral equation iteratively (with additional conditions, introduced *a priori condition*)
3	Howell & Walsh (1993)	Linearized → by removing one of the directional spectral product factors in the integrand as a spectral value at certain wave number vector. Then, it was modified into a matrix equation → was solved by a singular value decomposition
4	Hashimoto *et al* (1987, 1998, 2003)	Bayesian Method → more accurate (high accuracy), not in practical use. **Verification Bayesian Method** → more robust (validity & applicability) than Wyatt method, unfortunately time consuming iterative computation
5	Lukijanto *et al* (2009a, 2009b)	Modified Bayesian Method → more efficient and shorter computation time than original Bayesian Method (considered to be a practical method)

Bayesian Method (1)

A Bayesian Method, as one of the most accurate and reliable methods for estimating directional wave spectra (Hashimoto *et al*., 1987)

$S(f)\,\&\,G(\theta|f)$: *an exponential piecewise-constant function*

unknown variable: M, N

$$S(f,\theta)=\sum_{i=1}^{M}\sum_{j=1}^{N}\exp\left(x_{i,j}\right)\delta_{i,j}(f,\theta) \quad (13)^{**}$$

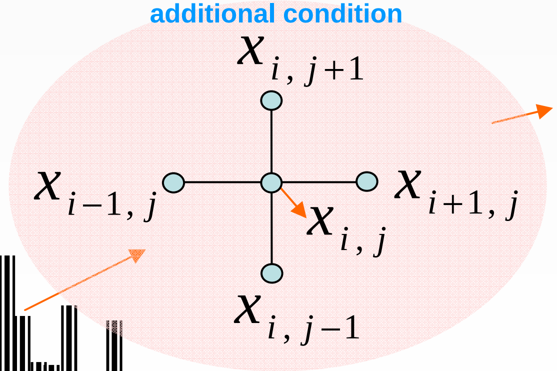
$x_{i,j}=\ln\left\{S(f_i,\theta_j)/\alpha\right\}$ ⇦ $\alpha=$ parameter introduced for normalizing magnitude $x_{i,j}$

$I=$ *number of segments* Δf *of* f

$J=$ *number of segments* $\Delta\theta$ *of* θ

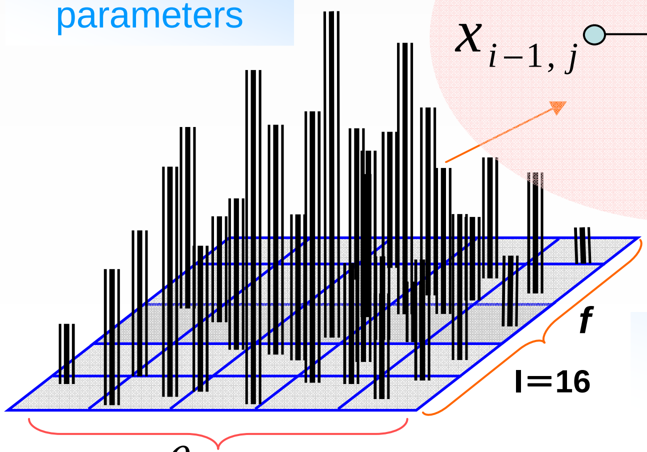
$$\delta_{i,j}(f,\theta)=\begin{cases}1: & f_{i-1}\le f\le f_i \text{ and } \theta_{j-1}\le\theta\le\theta_j\\ 0: & \text{otherwise}\end{cases} \quad (14)^{**}$$

additional condition

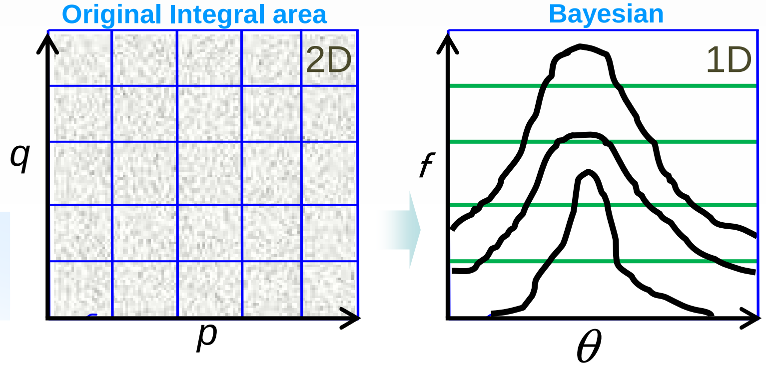


$$x_{i,j+1}+x_{i+1,j}+x_{i,j-1}+x_{i-1,j}-4x_{i,j}\approx 0$$

M X N
unknown parameters



L+(M X N)
Total Equation
Doppler Spectrum



Bayesian Method (2)

Fundamental Approach

Hyper parameter is introduced to consider the balance of the two requirements imposed on the estimate of the directional wave spectrum

- Maximizing the likelihood of the estimate &
- Maintaining the smoothness of the estimate

Number Equation > Number of Unknown Parameters

↓

To select the most suitable value of the Hyper parameter for the given Doppler spectra → **ABIC** (Akaike`s Bayesian Information Criterion) (Akaike, 1980)

↓

Numerical Examination

↓

Result

disadvantages

time consuming computation

Modified Bayesian Method (1)

A Modification of Bayesian Method

retains to the advantages of the Bayesian methods, and refers to the experience the previous MEP (Maximum Entropy Principle) research (Hashimoto & Kobune, 1986)

$S(f,\theta) \Rightarrow \sigma^{(2)}(\omega)$

characterized by exponential function having the power expressed by a Fourier series over the directional range $G(\theta|f)$, and assumed to be a piecewise over the frequency range $S(f)$

directional wave spectra density

directional wave spectrum

$$S(f_i,\theta) = \exp\left[a_0(f_i) + \sum_{k=1}^{K} \{ a_k(f_i)\cos k\theta + b_k(f_i)\sin k\theta \} \right]$$

unknown coefficients

$\tilde{\sigma}^{(2)}(\tilde{\omega}) \Rightarrow$ *restriction condition*

$$\tilde{\sigma}_l^{(2)} = F_l(\mathbf{X}) + \varepsilon_l$$

A line integral → must be performed along determined path due to the restrictions

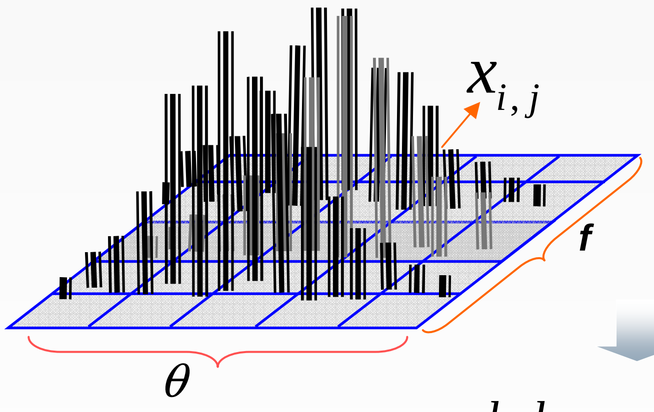
Modified Bayesian Method (2)

directional wave spectra density
directional wave spectrum

characterized by exponential function having the power expressed by a Fourier series over the directional range, and assumed to be a piecewise over the frequency range

$$S(f_i,\theta)=\exp\left[a_0(f_i)+\sum_{k=1}^{K}\{a_k(f_i)\cos k\theta+b_k(f_i)\sin k\theta\}\right]$$

unknown coefficients



unknown parameters:

M X (2 K+1)

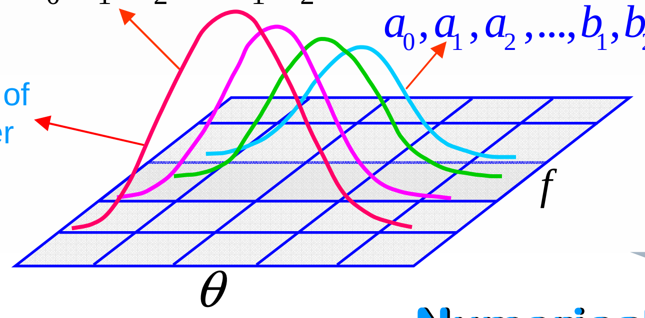
Number of *f* segments

Number of Fourier series

To reduce the unknown parameters (some time restriction condition is applied)

2 ways modifications

$a_0, a_1, a_2, \ldots, b_1, b_2, \ldots$



Should be connected of each shape parameter

Similar way Bayesian

Numerical Examination

$$\mathbf{X}_0=[a_k(f_i)\,\&\,b_k(f_i)]^t=0$$

Modified Bayesian Method (3)

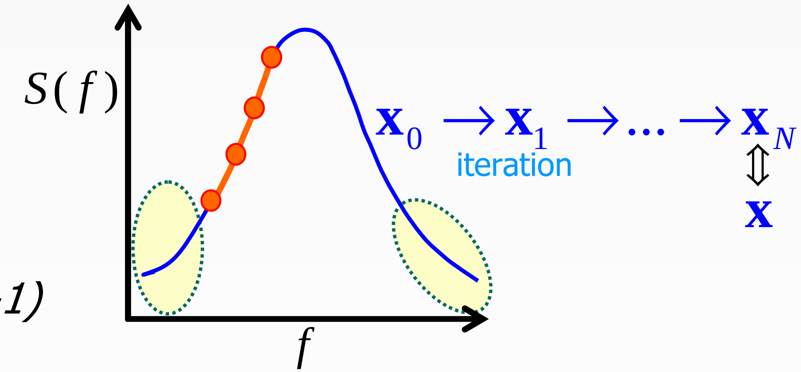
1

$S(f,\theta)$ is considered to be a smooth & continuous function and restriction conditions

$a_k(f_i)$ & $b_k(f_i)$ are locally continuous between adjacent frequencies in directional spectrum $S(f,\theta)$

unknown parameters:
(M-2) X (2 K+1)

L+(M-2) X (2 K+1) $\geq$ *MX (2 K+1)*



Additional Condition (1)

$$\left.\begin{matrix} a_k(f_{i+1}) - 2a_k(f_i) + a_k(f_{i-1}) \\ b_k(f_{i+1}) - 2b_k(f_i) + b_k(f_{i-1}) \end{matrix}\right\} \Rightarrow \begin{matrix} a_k(f_{i+1}) - a_k(f_i) \\ b_k(f_{i+1}) - b_k(f_i) \end{matrix}$$

$$W(\mathbf{X}) = \|\mathbf{AX} - \mathbf{B}\|^2 + u^2\|\mathbf{DX}\|^2$$

Results & Discussion (1)

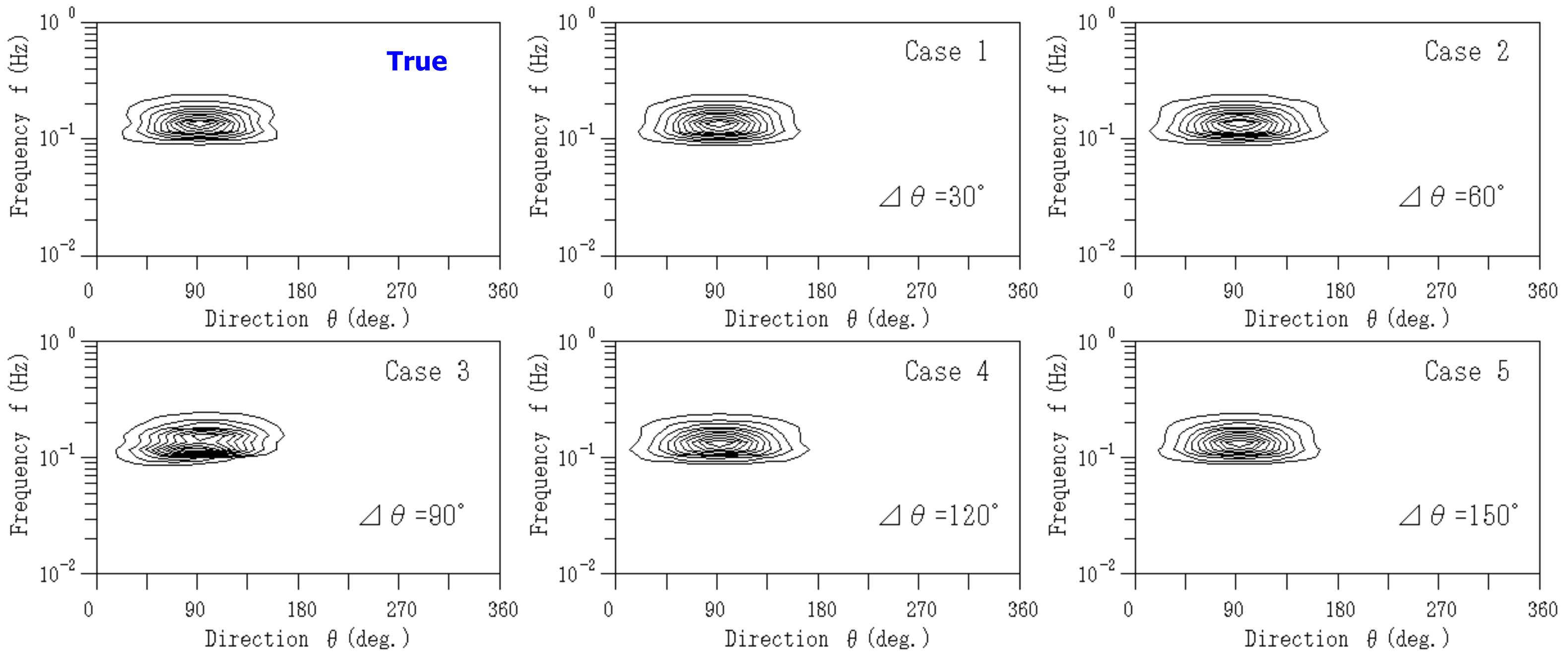
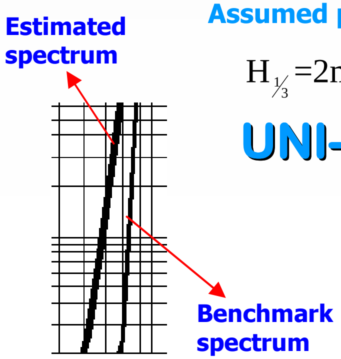


Fig.1. Excellent pattern of the estimated directional wave spectra where the dominant energy peaks of directional are assumed in the different frequency and different crossing angle of two beam axis.



Assumed parameters to be employed:

$H_{1/3}$=2m, $T_{1/3}$=8sec, α=90°, S=10m

UNI-DIRECTIONAL

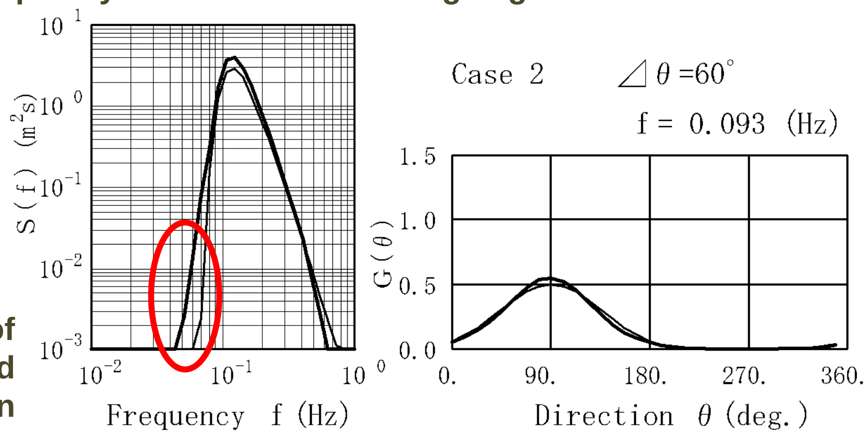


Fig.2. Comparison of frequency spectrum and direction function

Results & Discussion (2)

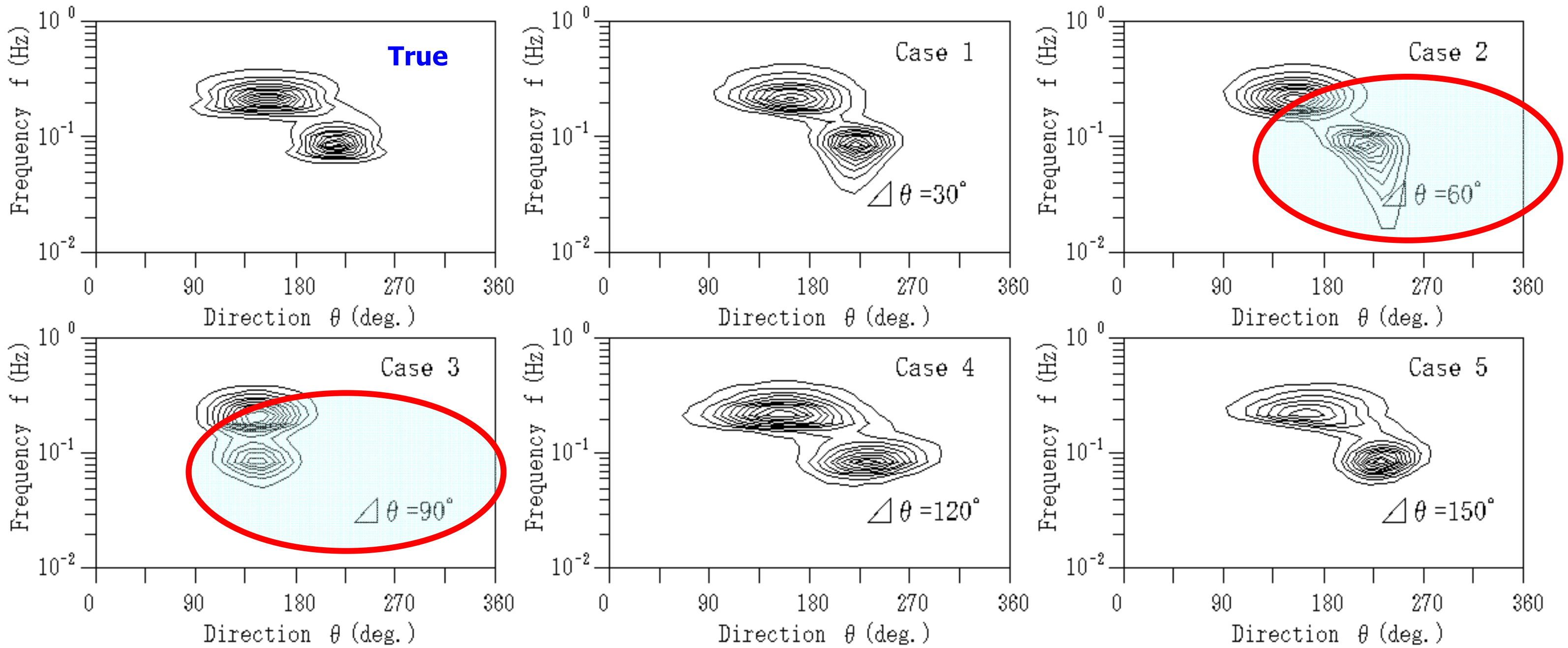
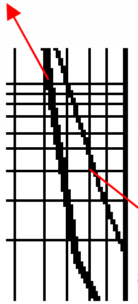


Fig.3. Excellent pattern of the estimated directional wave spectra where the dominant energy peaks of directional are assumed in the different frequency and different crossing angle of two beam axis.



Assumed parameters to be employed:

$H_{1/3}$=2m, $T_{1/3}$=5sec, α=150°, S=10m

$H_{1/3}$=1m, $T_{1/3}$=12sec, α=210°, S=25m

BI-DIRECTIONAL

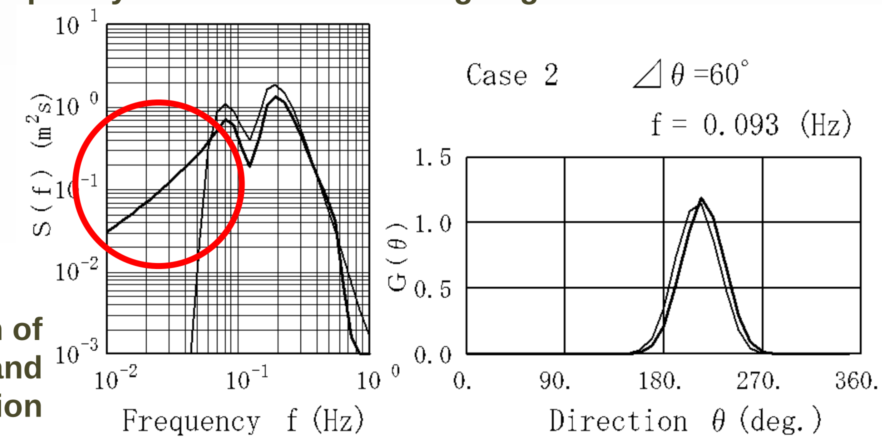


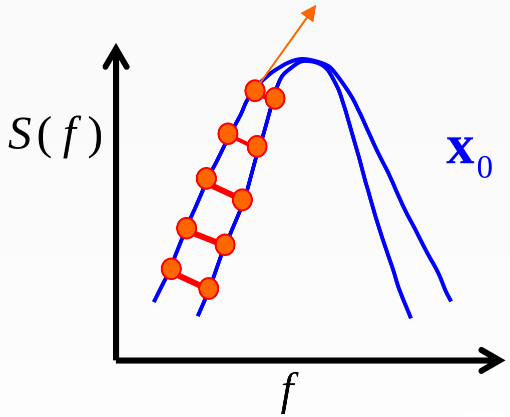
Fig.4. Comparison of frequency spectrum and direction function

Modified Bayesian Method (4)

2

$$W(\Delta\mathbf{X}) = \|\mathbf{A}\Delta\mathbf{X} - \mathbf{B}\|^2 + u^2\|\mathbf{D}\Delta\mathbf{X}\|^2$$

the difference between each
of continuous iteration



original equation of 1

$$\mathbf{x}_0 \rightarrow \mathbf{x}_1 \rightarrow \mathbf{x}_2 \rightarrow \mathbf{x}_N$$

$$\Updownarrow \quad \Updownarrow \quad \Updownarrow$$

$$\Delta\mathbf{x}_1 \quad \Delta\mathbf{x}_2 \quad \Delta\mathbf{x}_N$$

$$\mathbf{x} - \mathbf{x}_0 = \Delta\mathbf{x}_N$$

Additional Condition (2)

$$\left.\begin{array}{l} \Delta a_k(f_{i+1}) - 2\Delta a_k(f_i) + \Delta a_k(f_{i-1}) \\ \Delta b_k(f_{i+1}) - 2\Delta b_k(f_i) + \Delta b_k(f_{i-1}) \end{array}\right\} \quad (i = 2, \ldots, M-1)$$

$$\begin{array}{l} \Delta a_k(f_{i+1}) - \Delta a_k(f_i) \\ \Delta b_k(f_{i+1}) - \Delta b_k(f_i) \end{array} \quad (i = 1 \;\; and \;\; M)$$

Results & Discussion (3)

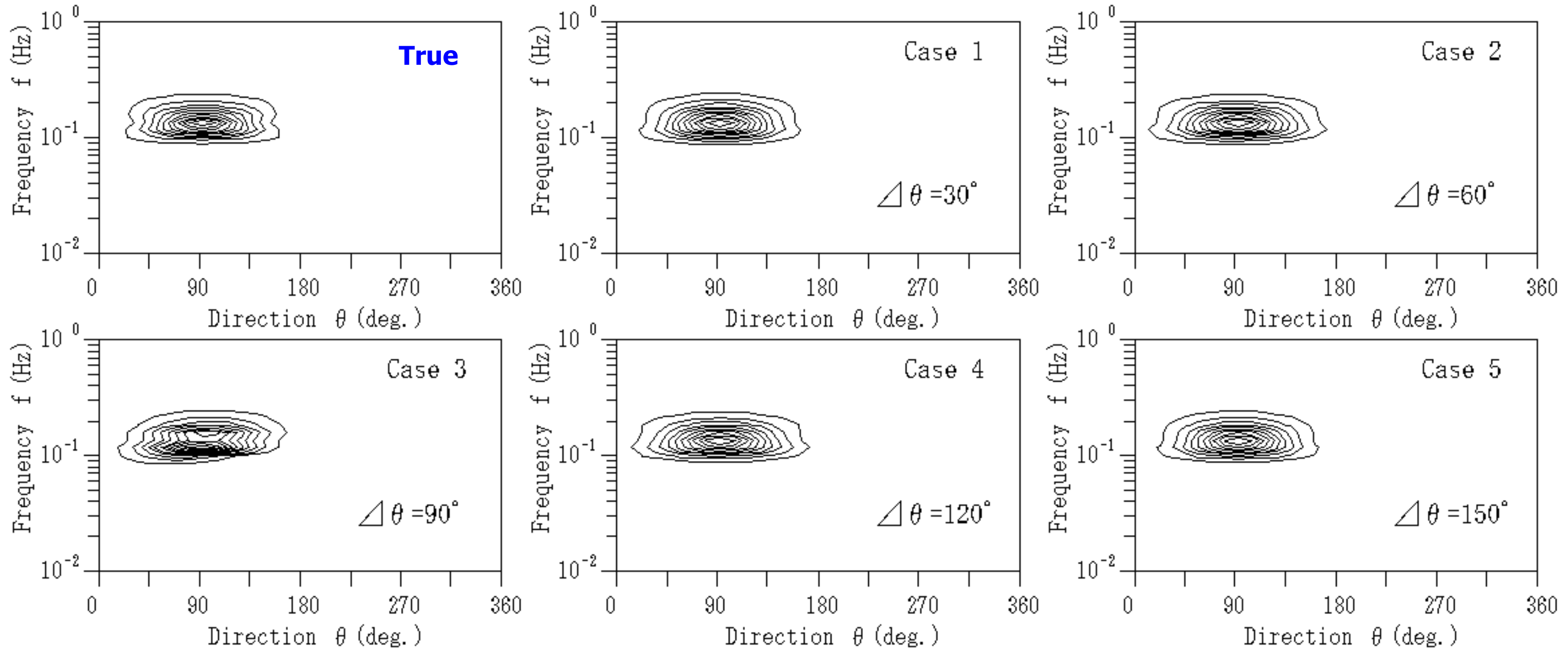
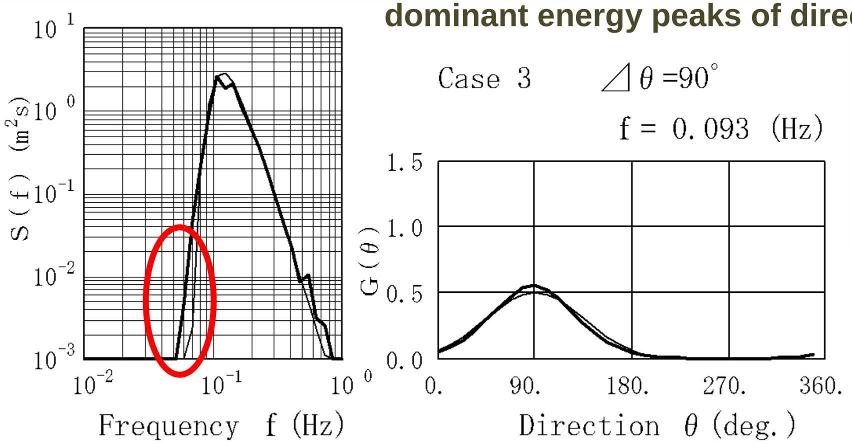


Fig.5. Excellent pattern of the estimated directional wave spectra where the dominant energy peaks of directional are assumed in the different frequency and different crossing angle of two beam axis.



Assumed parameters to be employed:

$H_{1/3}$=2m, $T_{1/3}$=8sec, α=90°, S=10m

UNI-DIRECTIONAL

Fig.6. Comparison of frequency spectrum and direction function

Results & Discussion (4)

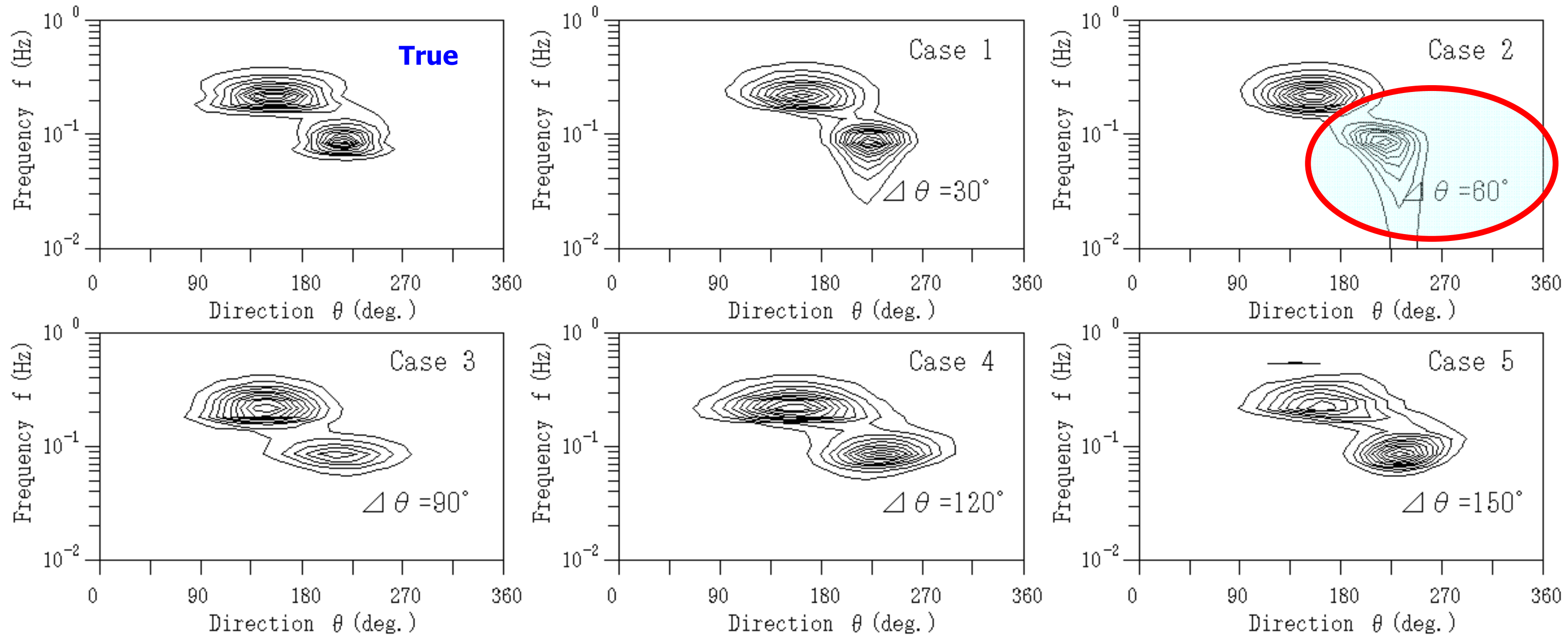
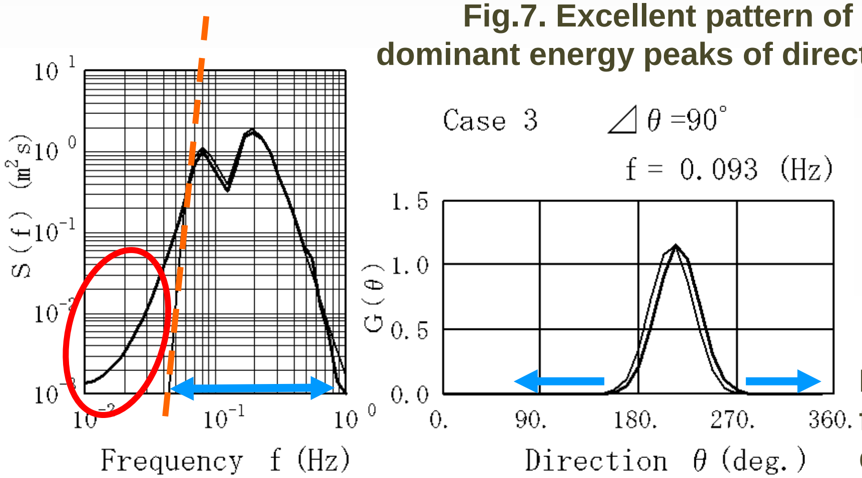


Fig.7. Excellent pattern of the estimated directional wave spectra where the dominant energy peaks of directional are assumed in the different frequency and different crossing angle of two beam axis.



Assumed parameters to be employed:

$$H_{1/3} = 2m,\ T_{1/3} = 5\sec,\ \alpha = 150^{\circ},\ S = 10m$$
$$H_{1/3} = 1m,\ T_{1/3} = 12\sec,\ \alpha = 210^{\circ},\ S = 25m$$

BI-DIRECTIONAL

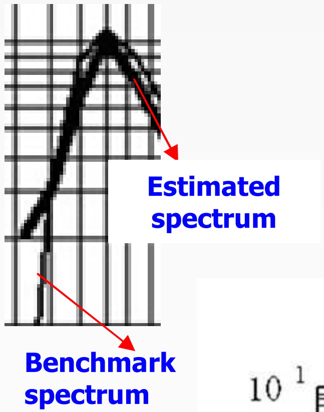
Fig.8. Comparison of frequency spectrum and direction function

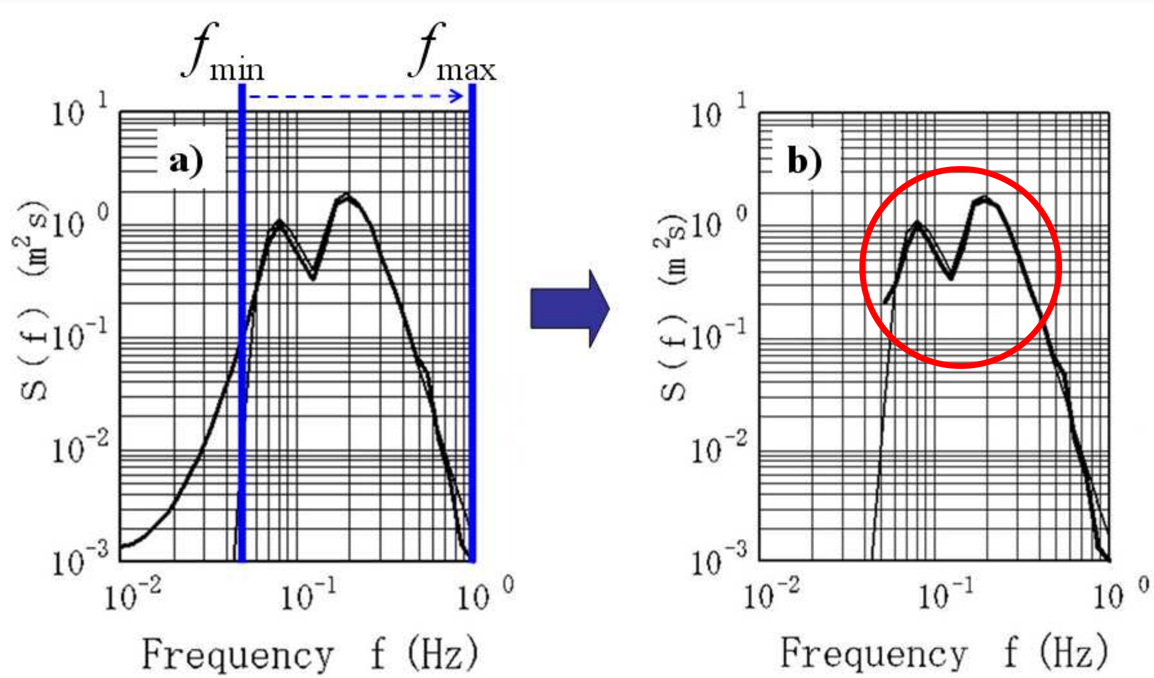
Improvements (1)

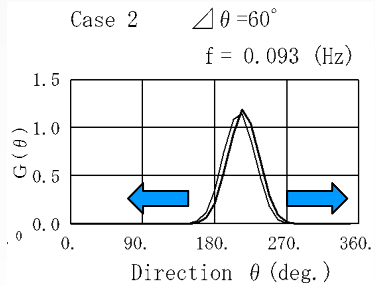
Due to the numerical instabilities

To assume the energy distribution of the specified frequency spectrum as a "*DEFINITION AREA*".

1







Improvements (2)

Due to the numerical instabilities

2 To add sufficient " *WHITE NOISE* " to disturb the Doppler spectrum

Bretschneider-Mitsuyasu type:

$$S(f) = 0.257\, H_{1/3}^2\, T_{1/3} \left(T_{1/3} f\right)^{-5} \exp\left[-1.03\left(T_{1/3} f\right)^{-4}\right]$$

$H_{1/3} = 3.0\text{m},\ T_{1/3} = 10.0\,\text{sec},\ S_{max} = 10$

Frequency Radar:

24.515 MHz

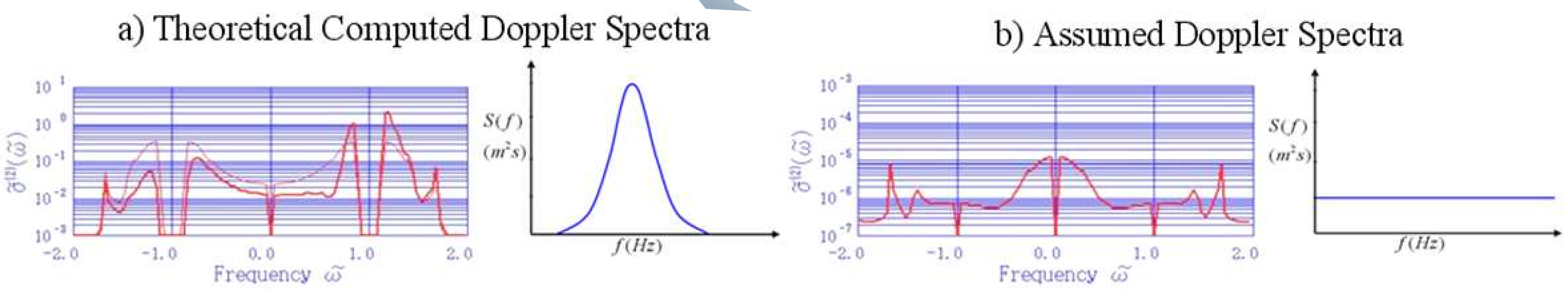


Figure. A schematic drawing of numerical simulation

Conclusions

- A new analysis of the Modification Bayesian Method has been presented that could estimate a good directional wave spectrum by HF oceanic radar. It is characterized by an exponential function having the power expressed by a Fourier series over the directional range, and assumed to be a piecewise-constant function over the frequency range.

- One important advantage of the new method (MBM) → it significantly reduces the storage capacity which requires less memory consumption (capacity) as well as shorter computation time.

- This confirms that compared to the previous works (BM) → the MBM may turn out to be not only accurate and reliable but also practical method for estimating directional wave spectra from HF radar

- Further research is underway to improve the numerical method and compare with other methods (i.e. Bayesian method, Wyatt method), as well as to verify the developed method with the actual field data.

Thank You Very Much for your attention