Modified Bayesian Method for Estimating Directional Wave Spectra from HF Radar Backscatter

by

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Introduction

The waves in ocean are multidirectional, and information about the directional distribution of wave energy is often required for the design of coastal and offshore structures. Sea surface current, ocean wave spectra and surface wind directions can be measured through frequency analysis of the radio wave backscattered from the ocean surface waves.

Schematic diagram of observation using HF ocean surface radar

The shore based radar transmits HF radio waves toward the sea surface and the radio waves are strongly backscattered by ocean surfaces waves with half the radio wave length (Bragg resonant scattering).
Introduction (2)

Theoretically, the directional wave spectra can be estimated from 1\textsuperscript{st} and 2\textsuperscript{nd} order Doppler spectra of HF radio wave with respect to the wave spectrum.

A method for estimating directional wave spectra from the data obtained by HF radar has not been established yet.

to develop the practical method for estimating directional wave spectra from HF backscattered information.

\[
\hat{S}(f, \theta) \xrightarrow{\text{Barrick's equation}} \sigma^{(2)}(\omega)
\]

An Identical Twin Experiment

Bayesian Method
Modified Bayesian Method for Estimating Directional Wave Spectra from HF Radar Backscatter

Fundamental Equations

Doppler Spectrum obtained from HF radar (Barrick, 1972):

HF \([\sigma(\omega) \approx \sigma^{(1)}(\omega) + \sigma^{(2)}(\omega)](1)\)

First Order Component:

\[\sigma^{(1)}(\omega) = 2^6 \pi k_0^4 \sum_{m = \pm 1} S(-2mk_0, 0) \delta(\omega - m\omega_B)\]

Crombie (1955) discovered that some HF signals recorded near the ocean had a Doppler shift due to "Bragg scattering".

Bragg's resonance condition:

\[k_1 + k_2 = -2k_0\]

Second Order Component:

\[\sigma^{(2)}(\omega) = 2^6 \pi k_0^4 \sum_{m_1m_2 = \pm 1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Gamma^2 S(m_1k_1)S(m_2k_2) \times \delta(\omega - m_1\sqrt{gk_1} - m_2\sqrt{gk_2})dpdq\]
The coupling coefficient is given:
\[
\Gamma = \left| \Gamma_H + \Gamma_E \right|^{(4)}
\]

\[
\Gamma_H = \frac{-i}{2} \left[ k_1 + k_2 - \frac{(k_1 k_2 - k_1 \cdot k_2) \left( \omega^2 + \omega_B^2 \right)}{m_1 m_2 \sqrt{k_1 k_2} \left( \omega^2 - \omega_B^2 \right)} \right]^{(5)}
\]

where:
- \( \omega_B \) = Bragg angular frequency
- \( k_1, k_2 \) = wave number vector

\[
\Gamma_{EM} = \frac{1}{2} \left[ \frac{(k_1 \cdot k_0)(k_2 \cdot k_0) / k_0^2 - 2k_1 \cdot k_2}{\sqrt{k_1 \cdot k_2} - k_0 \Delta} \right]^{(6)}
\]

Here \( \Delta \) refers to the normalized surface impedance derived by Barrick (1972). The absolute value of which is small enough \( \rightarrow \) negligible.
**Fundamental Equations**

**Normalized Doppler Spectrum**

\[ \frac{\lambda}{2} \] \( \sigma^{(1)} \)

\[ \frac{\lambda}{2} \] \( \sigma^{(2)} \)

**PROBLEMS?**

The integral is restricted to being solved only on specific lines defined by \( \delta \)

Thus it must satisfy:

\[ \omega - m_1 \sqrt{g k_1} - m_2 \sqrt{g k_2} = 0 \]

The digitization of this equation is complicated (incomplete inverse problem) →

*number of unknown parameters >> that of equations*

\[ (\eta > 1.0)(\omega > \omega_B) \]

\[ \eta = 2^\frac{1}{2} \]

\[ |\omega| > \omega_B (i.e. m = m') \]

\[ (\eta < 1.0)(\omega < \omega_B) \]

\[ |\omega| < \omega_B (i.e. m = -m') \]
Modified Bayesian Method for Estimating Directional Wave Spectra from HF Radar Backscatter

\[ \sigma^{(2)}(\omega) = 2^6 \pi K_0^4 \sum_{m_1m_2=\pm 1} \int \int |\Gamma|^2 S(m_1k_1)S(m_2k_2) \times \delta(\omega-m_1\sqrt{gk_1}-m_2\sqrt{gk_2}) \, dp \, dq \]  \hspace{1cm} (3)

For convenience, the parameters are nondimensionalized by \( \omega_l \) and \( 2k_0 \) as follows:

\[ \tilde{\omega} = \omega / \omega_B \quad \tilde{k} = k / (2k_0) \quad \Gamma = \Gamma / (2k_0) \quad S(\tilde{k}) = (2k_0)^4 S(k) \] \hspace{1cm} (7)

The integration 2\(^{nd}\) order with respect to the p & q can be transformed into a single variable since the integrand include \( \delta \). (Lippa & Barrick, 82) if the wave propagation direction \( \theta_1 \) of the wave number vector \( k_1 \) is adopted as a single independent variable for the integration.

\[ \tilde{\sigma}^{(2)}(\tilde{\omega}) = \int_0^{\theta_L} G(\theta_1, \tilde{\omega}) \, d\theta_1 \] \hspace{1cm} (8)

where

\[ G(\theta, \tilde{\omega}) = 16\pi \left| \Gamma \right|^2 \left\{ \tilde{S}(m_1\tilde{k}_1) \tilde{S}(m_2\tilde{k}_2) + \tilde{S}(m_1\tilde{k}_1 \ast) \tilde{S}(m_2\tilde{k}_2 \ast) \right\} y^3 \left| dy / dh \right| \] \hspace{1cm} (9)

Theoretically

\[ S(k) = \frac{S(f, \theta)}{2^5 \pi^4 f^3} \] \hspace{1cm} (12)
Recent Researches

Efforts to extract HF radar for directional wave spectra

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<td>Wyatt (1990)</td>
<td>Relaxation method → assumed wave spectral form at HF &amp; the nonlinear integral equation is modified into a linear integral equation which can be solved by this method</td>
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<td>2</td>
<td>Hisaki (1996)</td>
<td>Development of linearization method → to solve the nonlinear integral equation iteratively (with additional conditions, introduced a priori condition)</td>
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<td>3</td>
<td>Howell &amp; Walsh (1993)</td>
<td>Linearized → by removing one of the directional spectral product factors in the integrand as a spectral value at certain wave number vector. Then, it was modified into a matrix equation → was solved by a singular value decomposition</td>
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<td>5</td>
<td>Lukijanto et al (2009a, 2009b)</td>
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Modified Bayesian Method for Estimating Directional Wave Spectra from HF Radar Backscatter

A Bayesian Method, as one of the most accurate and reliable methods for estimating directional wave spectra (Hashimoto et al., 1987)

\[
S(f, \theta) = \sum_{i=1}^{M} \sum_{j=1}^{N} \exp \left( x_{i,j} \right) \delta_{i,j}(f, \theta)
\]

unknown variable

\[
x_{i,j} = \ln \left\{ S(f_i, \theta_j) / \alpha \right\} \quad \alpha = \text{parameter introduced for normalizing magnitude } x_{i,j}
\]

\[
\delta_{i,j}(f, \theta) = \begin{cases} 
1: & f_{i-1} \leq f \leq f_i \text{ and } \theta_{j-1} \leq \theta \leq \theta_j \\
0: & \text{otherwise}
\end{cases}
\]

additional condition

\[
x_{i,j+1} + x_{i+1,j} + x_{i,j-1} + x_{i-1,j} - 4x_{i,j} \approx 0
\]

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Hyper parameter is introduced to consider the balance of the two requirements imposed on the estimate of the directional wave spectrum:

- Maximizing the likelihood of the estimate &
- Maintaining the smoothness of the estimate

\[ \text{Number Equation} > \text{Number of Unknown Parameters} \]

To select the most suitable value of the Hyper parameter for the given Doppler spectra, we use \( \text{ABIC} \) (Akaike's Bayesian Information Criterion) (Akaike, 1980).

\[ \text{Result} \]

- \text{disadvantages}
  - time consuming computation
A Modification of Bayesian Method

retains to the advantages of the Bayesian methods, and refers to the experience the previous MEP (Maximum Entropy Principle) research (Hashimoto & Kobune, 1986)

\[ S(f, \theta) \Rightarrow \sigma^{(2)}(\omega) \]

directional wave spectra density

characterized by exponential function having the power expressed by a Fourier series over the directional range \( G(\theta, f) \), and assumed to be a piecewise over the frequency range \( S(f) \)

\[ S(f_i, \theta) = \exp \left[ a_0(f_i) + \sum_{k=1}^{K} \left\{ a_k(f_i) \cos k\theta + b_k(f_i) \sin k\theta \right\} \right] \]

unknown coefficients

\[ \tilde{\sigma}^{(2)}(\tilde{\omega}) \Rightarrow \text{restriction condition} \]

A line integral \( \Rightarrow \text{must be performed} \)
along determined path due to the restrictions
Modified Bayesian Method (2)

The directional wave spectra density $S(f_i, \theta)$ is given by:

$$S(f_i, \theta) = \exp \left[ a_0(f_i) + \sum_{k=1}^{K} \left\{ a_k(f_i) \cos k\theta + b_k(f_i) \sin k\theta \right\} \right]$$

where:
- $a_0, a_1, a_2, b_1, b_2, \ldots$ are unknown coefficients.
- $\theta$ is the directional angle.
- $f_i$ is the frequency.

Characterized by an exponential function having the power expressed by a Fourier series over the directional range, and assumed to be piecewise over the frequency range.

Unknown parameters:
- $M \times (2(K+1))$
- Number of $f$ segments
- Number of Fourier series

To reduce the unknown parameters (some time restriction condition is applied)

2 ways modifications

Numerical Examination

$$X_0 = [a_k(f_i) & b_k(f_i)] = 0$$
Modified Bayesian Method (3)

$S(f, \theta)$ is considered to be a smooth & continuous function and restriction conditions

$a_k(f_i) & b_k(f_i)$ are locally continuous between adjacent frequencies in directional spectrum $S(f, \theta)$

unknown parameters: 
$(M-2) \times (2K+1)$

$L+(M-2) \times (2K+1) \geq MX (2K+1)$

Additional Condition (1)

\[
\begin{align*}
&a_k(f_{i+1}) - 2a_k(f_i) + a_k(f_{i-1}) \\
&b_k(f_{i+1}) - 2b_k(f_i) + b_k(f_{i-1})
\end{align*}
\]

\[
\begin{align*}
&a_k(f_{i+1}) - a_k(f_i) \\
&b_k(f_{i+1}) - b_k(f_i)
\end{align*}
\]

\[
W(X) = \|AX - B\|^2 + u^2 \|DX\|^2
\]
Results & Discussion

Fig. 1. Excellent pattern of the estimated directional wave spectra where the dominant energy peaks of directional are assumed in the different frequency and different crossing angle of two beam axis.

Assumed parameters to be employed:

\[ H_s = 2m, T_s = 8\text{sec}, \alpha = 90^\circ, S = 10m \]

UNI-DIRECTIONAL

Fig. 2. Comparison of frequency spectrum and direction function
Results & Discussion

Fig. 3. Excellent pattern of the estimated directional wave spectra where the dominant energy peaks of directional are assumed in the different frequency and different crossing angle of two beam axis.

Assumed parameters to be employed:
- $H_s = 2m$, $T_s = 5sec$, $\alpha = 150^\circ$, $S = 10m$
- $H_s = 1m$, $T_s = 12sec$, $\alpha = 210^\circ$, $S = 25m$

Fig. 4. Comparison of frequency spectrum and direction function

BI-DIRECTIONAL
Modified Bayesian Method

\[ W(\Delta X) = \| A\Delta X - B \|^2 + u^2 \| D\Delta X \|^2 \]

the difference between each of continuous iteration

\[
\begin{align*}
\Delta a_k(f_{i+1}) - 2\Delta a_k(f_i) + \Delta a_k(f_{i-1}) \\
\Delta b_k(f_{i+1}) - 2\Delta b_k(f_i) + \Delta b_k(f_{i-1})
\end{align*}
\]

\[
\begin{align*}
(i = 2, \ldots, M - 1) & \quad \Rightarrow \quad \Delta a_k(f_{i+1}) - \Delta a_k(f_i) \\
(i = 1 \text{ and } M) & \quad \Rightarrow \quad \Delta b_k(f_{i+1}) - \Delta b_k(f_i)
\end{align*}
\]
Results & Discussion

Fig. 5. Excellent pattern of the estimated directional wave spectra where the dominant energy peaks of directional are assumed in the different frequency and different crossing angle of two beam axis.

Assumed parameters to be employed:
\( H_x = 2 \) m, \( T_x = 8 \) sec, \( \alpha = 90^\circ \), \( S = 10 \) m

UNI-DIRECTIONAL

Fig. 6. Comparison of frequency spectrum and direction function
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Results & Discussion

Fig. 7. Excellent pattern of the estimated directional wave spectra where the dominant energy peaks of directional are assumed in the different frequency and different crossing angle of two beam axis.

Assumed parameters to be employed:
- $H_{\lambda} = 2m, T_{\lambda} = 5$ sec, $\alpha = 90^\circ, S = 10m$
- $H_{\lambda} = 1m, T_{\lambda} = 12$ sec, $\alpha = 210^\circ, S = 25m$

Fig. 8. Comparison of frequency spectrum and direction function
Improvements (1)

Due to the numerical instabilities

To assume the energy distribution of the specified frequency spectrum as a “DEFINITION AREA”.

Benchmark spectrum

Estimated spectrum

Case 2
\[ \triangle \theta = 60^\circ \]
\[ f = 0.093 \text{ (Hz)} \]
Improvements (2)

Due to the numerical instabilities

To add sufficient “WHITE NOISE” to disturb the Doppler spectrum

\[
S(f) = 0.257 H_{1/3}^2 T_{1/3}^{-5} \exp\left[-1.03 \left( T_{1/3} f \right)^{-4}\right]
\]

Frequency Radar: 24.515 MHz

\begin{align*}
H_{1/3} & = 3.0\text{m}, \quad T_{1/3} = 10.0\text{sec}, \quad S_{\text{max}} = 10
\end{align*}

Figure. A schematic drawing of numerical simulation
Conclusions

- A new analysis of the Modification Bayesian Method has been presented that could estimate a good directional wave spectrum by HF oceanic radar. It is characterized by an exponential function having the power expressed by a Fourier series over the directional range, and assumed to be a piecewise-constant function over the frequency range.

- One important advantage of the new method (MBM) is that it significantly reduces the storage capacity which requires less memory consumption (capacity) as well as shorter computation time.

- This confirms that compared to the previous works (BM), the MBM may turn out to be not only accurate and reliable but also practical method for estimating directional wave spectra from HF radar.

- Further research is underway to improve the numerical method and compare with other methods (i.e. Bayesian method, Wyatt method), as well as to verify the developed method with the actual field data.
Thank You Very Much for your attention